

## Integrals involving Roots:

$$\int \frac{x+2}{\sqrt[3]{x-3}} dx$$

$$\begin{aligned} & \cancel{x-3} \cdot u \\ & \cancel{dx} = du \end{aligned}$$

$$(u)^3 = (\sqrt[3]{x-3})^3$$

$$u^3 = x-3$$

$$x = u^3 + 3$$

$$dx = 3u^2 du$$

$$\int \frac{(u^3+3)+2}{u} \cdot 3u^2 du$$

$$\int 3u^4 + 15u du$$

$$= \frac{3u^5}{5} + \frac{15u^2}{2} + C$$

$$= \frac{3}{5} \left[ (x-3)^{5/3} \right] + \frac{15}{2} \left[ (x-3)^{2/3} \right] + C$$

$$= \frac{3}{5} (x-3)^{5/3} + \frac{15}{2} (x-3)^{2/3} + C$$

$$\textcircled{2} \int \frac{2}{x-3\sqrt{x+10}} dx$$

$$u = \sqrt{x+10}$$

$$u^2 = x+10$$

$$x = u^2 - 10$$

$$dx = 2u du$$

$$\int \frac{2 \cdot 2u du}{u^2 - 10 - 3u}$$

$$\int \frac{4u du}{u^2 - 3u - 10} = \int \frac{4u du}{(u-5)(u+2)}$$

$$= \int \frac{20}{7(u-5)} + \frac{8}{7(u+2)} du$$

$$= \frac{20}{7} \ln|u-5| + \frac{8}{7} \ln|u+2|$$

$$= \frac{20}{7} \ln|\sqrt{x+10}-5| + \frac{8}{7} \ln|\sqrt{x+10}+2| + C \quad \text{or } C = \frac{8}{7}$$

$$\frac{4u}{(u-5)(u+2)} = \frac{A(u+2)}{u-5} + \frac{B(u-5)}{u+2}$$

$$4u = A(u+2) + B(u-5)$$

$$(A+B=4) \quad \begin{cases} 5A + 5B = 20 \\ 2A - 5B = 0 \end{cases}$$

$$2A - 5B = 0$$

$$\begin{array}{r} 7A = 20 \\ A = 20/7 \end{array}$$

## Integrals involving Quadratics

Formula  $\int \frac{x}{x^2 \pm a} dx = \frac{1}{2} \ln |x^2 \pm a| + C$

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right)$$

$$\int \frac{2x+3}{4x^2+12x-1} dx$$

$$\frac{1}{4} \int \frac{du}{u}$$

$$= \frac{1}{4} \ln |4x^2+12x-1| + C$$

$$u = 4x^2 + 12x - 1$$

$$du = 8x + 12 dx$$

$$\frac{du}{4} = \frac{4(2x+3)}{4} dx$$

$$\int \frac{3x-1}{x^2+10x+28} dx$$

$$\int \frac{3x-1}{(x+5)^2+3} dx$$

$$\int \frac{3(u-5)-1}{u^2+3} du$$

$$\int \frac{3u-16}{u^2+3} du = \int \frac{3u}{u^2+3} - \frac{16}{u^2+3} du$$

$$= \frac{3}{2} \ln |u^2+3| - \frac{16}{\sqrt{3}} \tan^{-1} \left( \frac{u}{\sqrt{3}} \right) + C$$

$$= \frac{3}{2} \ln |(x+5)^2+3| - \frac{16}{\sqrt{3}} \tan^{-1} \left( \frac{x+5}{\sqrt{3}} \right) + C$$

$$\begin{aligned} x^2+10x+28 &= x^2+10x+25-25+28 \\ &= (x+5)^2+3 \end{aligned}$$

$$u = x+5 \Rightarrow x = u-5$$
$$du = dx$$

Hi

hello